Modification of Kawai Model about the Mixing of the Pseudoscalar Mesons

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Abstract

The Kawai model describing the glueball-quarkonia mixing is modified. The mixing of η, η' and $\eta(1410)$ is re-investigated based on the modified Kawai model. The glueball-quarkonia content of the three states is determined from a fit to the data of the electromagnetic decays involving η, η'. Some predictions about the electromagnetic decays involving $\eta(1410)$ are presented.

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I. INTRODUCTION

The 0^- ground state nonet is one of the best established $q\bar{q}$ multiplets. The isotriplet $\pi(1300)$ and the isodoublet $K(1460)$ of the 0^- first radial excitation have been established [1] and the $\eta(1295)$ can be identified as the first radial excitation of η [2,3]. In addition, $\eta(1440)$ has been resolved into two states: $\eta(1490)$ and $\eta(1410)$ [4–6](let η'' stand for the $\eta(1410)$ below). The former has been interpreted as the mainly $s\bar{s}$ radial excitation of η' [2,3,7,8] and the latter seems a spurious state, which is argued to be a mainly glueball, possibly mixed with $q\bar{q}$ states [7,8].

In general, states with the same isospin-spin-parity IJ^{PC} and additive quantum numbers can mix. The fact that $M_{\eta(1295)} \approx M_{\pi(1300)}$ [9] implies that $\eta(1490)$ and $\eta(1295)$ are almost ideal mixing. Therefore, the possibility of mixing of ground states and radial excitations can be ignored, then one can focus on the mixing of η, η' and η''. The mixing of η, η' and η'' has been discussed in Ref. [10] based on the mass-squared matrix

$$M^2 = \begin{pmatrix} M_N^2 + rA_1 & \sqrt{r}A_1 & \sqrt{r}A_2 \\ \sqrt{r}A_1 & M_S^2 + A_1 & A_2 \\ \sqrt{r}A_2 & A_2 & M_{G_0}^2 + A_3 \end{pmatrix} \tag{1}$$

with the $|N\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$, $|S\rangle = |s\bar{s}\rangle$ and $|G_0\rangle = |gg\rangle$ basis[1], where M_N, M_S and M_{G_0} are the masses of primitive (unmixed) $|N\rangle$, $|S\rangle$ and $|G_0\rangle$, respectively; A_1, A_2, $A_3 = A_2^2/A_1$ describe the transitions between strangeonium and strangeonium, between strangeonium and gluonium, and between gluonium and gluonium, respectively. r describes the effect of flavor-dependent transition taking into account the possibility that the nonstrange quarkonia and strange quarkonia system have the different wave functions at the origin as the result

[1]Here, A_1, A_2, A_3 and r respetively correspond to λ_S^2, $\lambda_S\lambda_G$, λ_G^2 and $2\lambda_N^2/\lambda_S^2$ employed in Ref. [10]

of the different mass (in $SU(3)$ limit, $r = 2$). The eigenvalues of M^2 are M^2_η, $M^2_{\eta'}$ and $M^2_{\eta''}$, the masses square of the physical states η, η' and η'', respectively.

However, we believe this mixing model should be modified for the pseudoscalar mesons. In Ref [10], it is pointed out that $M^2_{G_0} \simeq 2\langle k_T^2\rangle$ for a digluon-ball, where $\langle k_T^2\rangle$ is the transverse momentum fluctuation of the constituent gluons, and that A_3 is considered as the additional contribution to the matrix M^2 due to the transition between gluonium and gluonium. In the viewpoint of lattice QCD, the value of M_{G_0} would be related to the prediction about the mass of the pseudoscalar glueball in quenched approximation since A_3 at least can contain the contribution arising from the transitions of $|G_0\rangle$ to a quark pair and back to $|G_0\rangle$. However, based on Eq. (1) M_{G_0} is determined to be the value of about 1.3 GeV [10], which is obviously inconsistent with 2.56 ± 0.13 GeV [11], the mass of pseudoscalar glueball predicted by lattice QCD in quenched approximation. Furthermore, in the presence of $A_3 = A_2^2/A_1$, if one restricts M_{G_0} to be comparable with the prediction given by lattice QCD in quenched approximation for the pseudoscalar glueball mass in the matrix M^2 (i.e., $M_{G_0} > 2$ GeV) and assumes the eigenvalues of M^2 are the masses square of η, η' and η'', respectively, based on Eqs. (6)$\sim$(8), one can have $A_2^2 < 0$ which would cause the matrix M^2 to be a non-hermitian matrix. In fact, in the pseudoscalar mesons sector, A_1, A_2 and A_3 should be nonperturbative effect, and the relation of A_1, A_2 and A_3 is completely unknown in principle, therefore there is no convincing reason to expect that the relation of A_1, A_2 and A_3 should behave as $A_3 = A_2^2/A_1$. In this work, we shall relate M_{G_0} to the prediction of the pseudoscalar glueball mass given by lattice QCD in quenched approximation and consider A_3 as a free parameter describing the sum of all fermion-loop corrections to the quenched prediction of the pseudoscalar glueball mass.

II. MIXING OF η, η' AND η'' BASED ON THE MODIFIED KAWAI MODEL

If A_3 is considered as a free parameter rather than $A_3 = A_2^2/A_1$ as usual in Ref. [10], diagonolizing the matrix M^2, one can get

$$U M^2 U^\dagger = \begin{pmatrix} M^2_{\eta''} & 0 & 0 \\ 0 & M^2_{\eta'} & 0 \\ 0 & 0 & M^2_\eta \end{pmatrix}, \tag{2}$$

where

$$U = \begin{pmatrix} x_{\eta''} & y_{\eta''} & z_{\eta''} \\ x_{\eta'} & y_{\eta'} & z_{\eta'} \\ x_\eta & y_\eta & z_\eta \end{pmatrix} \tag{3}$$

and

$$\begin{aligned} x_i &= \sqrt{r}(M_i^2 - M_S^2)(A_2^2 - A_1A_3 + A_1M_i^2 - A_1M_{G_0}^2)/f_i, \\ y_i &= (M_i^2 - M_N^2)(A_2^2 - A_1A_3 + A_1M_i^2 - A_1M_{G_0}^2)/f_i, \\ z_i &= (M_i^2 - M_N^2)(M_i^2 - M_S^2)A_2/f_i, \end{aligned} \tag{4}$$

with

$$\begin{aligned} f_i = \{ & r[(M_i^2 - M_S^2)(A_2^2 - A_1A_3 + A_1M_i^2 - A_1M_{G_0}^2)]^2 \\ & +[(M_i^2 - M_N^2)(A_2^2 - A_1A_3 + A_1M_i^2 - A_1M_{G_0}^2)]^2 \\ & +[(M_i^2 - M_N^2)(M_i^2 - M_S^2)A_2]^2\}^{\frac{1}{2}}, \end{aligned}$$

i=η'', η' and η. The physical states $|\eta\rangle$, $|\eta'\rangle$ and $|\eta''\rangle$ can be read as

$$\begin{pmatrix} |\eta''\rangle \\ |\eta'\rangle \\ |\eta\rangle \end{pmatrix} = U \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G_0\rangle \end{pmatrix}. \tag{5}$$

From Eq. (2), one can have

$$M^2_{\eta''}M^2_{\eta'}M^2_\eta = (A_3 + M^2_{G_0})(A_1M_N^2 + M_N^2M_S^2 + A_1M_S^2 r) - A_2^2(M_N^2 + M_S^2 r), \tag{6}$$

$$\begin{aligned} M^2_{\eta''}M^2_{\eta'} + M^2_{\eta''}M^2_\eta + M^2_{\eta'}M^2_\eta = {} & A_3M_N^2 + M^2_{G_0}M_N^2 + A_3M_S^2 + M^2_{G_0}M_S^2 + M_N^2M_S^2 - A_2^2(1+r) \\ & + A_1(A_3 + M^2_{G_0} + M_N^2 + A_3 r + M^2_{G_0} r + M_S^2 r), \end{aligned} \tag{7}$$

$$M^2_{\eta''} + M^2_{\eta'} + M^2_\eta = M_N^2 + M_S^2 + M^2_{G_0} + rA_1 + A_1 + A3. \tag{8}$$

For the electromagnetic decays involving η, η' and η'', based on Eq. (5), performing an elementary $SU(3)$ calculation [12–14], one can obtain the following equations:

$$\frac{\Gamma(\eta\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{1}{9}\left(\frac{M_\eta}{M_{\pi^0}}\right)^3(5x_\eta+\sqrt{2}y_\eta)^2, \tag{9}$$

$$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{1}{9}\left(\frac{M_{\eta'}}{M_{\pi^0}}\right)^3(5x_{\eta'}+\sqrt{2}y_{\eta'})^2, \tag{10}$$

$$\frac{\Gamma(\rho\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\left[\frac{(M_\rho^2-M_\eta^2)M_\omega}{(M_\omega^2-M_{\pi^0}^2)M_\rho}\right]^3x_\eta^2, \tag{11}$$

$$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=3\left[\frac{(M_{\eta'}^2-M_\rho^2)M_\omega}{(M_\omega^2-M_{\pi^0}^2)M_{\eta'}}\right]^3x_{\eta'}^2, \tag{12}$$

$$\frac{\Gamma(\phi\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{4}{9}\frac{m_u^2}{m_s^2}\left[\frac{(M_\phi^2-M_\eta^2)M_\omega}{(M_\omega^2-M_{\pi^0}^2)M_\phi}\right]^3y_\eta^2, \tag{13}$$

$$\frac{\Gamma(\phi\to\eta'\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{4}{9}\frac{m_u^2}{m_s^2}\left[\frac{(M_\phi^2-M_{\eta'}^2)M_\omega}{(M_\omega^2-M_{\pi^0}^2)M_\phi}\right]^3y_{\eta'}^2, \tag{14}$$

$$\frac{\Gamma(J/\psi\to\rho\eta)}{\Gamma(J/\psi\to\omega\pi^0)}=\left[\frac{\sqrt{[M_{J/\psi}^2-(M_\rho+M_\eta)^2][M_{J/\psi}^2-(M_\rho-M_\eta)^2]}}{\sqrt{[M_{J/\psi}^2-(M_\omega+M_{\pi^0})^2][M_{J/\psi}^2-(M_\omega-M_{\pi^0})^2]}}\right]^3x_\eta^2, \tag{15}$$

$$\frac{\Gamma(J/\psi\to\rho\eta')}{\Gamma(J/\psi\to\omega\pi^0)}=\left[\frac{\sqrt{[M_{J/\psi}^2-(M_\rho+M_{\eta'})^2][M_{J/\psi}^2-(M_\rho-M_{\eta'})^2]}}{\sqrt{[M_{J/\psi}^2-(M_\omega+M_{\pi^0})^2][M_{J/\psi}^2-(M_\omega-M_{\pi^0})^2]}}\right]^3x_{\eta'}^2, \tag{16}$$

$$\frac{\Gamma(\eta''\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{1}{9}\left(\frac{M_{\eta''}}{M_{\pi^0}}\right)^3(5x_{\eta''}+\sqrt{2}y_{\eta''})^2, \tag{17}$$

$$\frac{\Gamma(\eta''\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=3\left[\frac{(M_{\eta''}^2-M_\rho^2)M_\omega}{(M_\omega^2-M_{\pi^0}^2)M_{\eta''}}\right]^3x_{\eta''}^2, \tag{18}$$

$$\frac{\Gamma(\eta''\to\omega\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{1}{3}\left[\frac{(M_{\eta''}^2-M_\omega^2)M_\omega}{(M_\omega^2-M_{\pi^0}^2)M_{\eta''}}\right]^3x_{\eta''}^2, \tag{19}$$

$$\frac{\Gamma(\eta''\to\phi\gamma)}{\Gamma(\omega\to\pi^0\gamma)}=\frac{4}{9}\frac{m_u^2}{m_s^2}\left[\frac{(M_{\eta''}^2-M_\phi^2)M_\omega}{(M_\omega^2-M_{\pi^0}^2)M_{\eta''}}\right]^3y_{\eta''}^2, \tag{20}$$

$$\frac{\Gamma(J/\psi\to\rho\eta'')}{\Gamma(J/\psi\to\omega\pi^0)}=\left[\frac{\sqrt{[M_{J/\psi}^2-(M_\rho+M_{\eta''})^2][M_{J/\psi}^2-(M_\rho-M_{\eta''})^2]}}{\sqrt{[M_{J/\psi}^2-(M_\omega+M_{\pi^0})^2][M_{J/\psi}^2-(M_\omega-M_{\pi^0})^2]}}\right]^3x_{\eta''}^2, \tag{21}$$

where M_ρ, M_ω, M_ϕ and $M_{J/\psi}$ are the masses of ρ, ω, ϕ and J/ψ, respectively; m_u and m_s are the masses of the constituent quark u and d, respectively.

III. FIT RESULTS

In Eq. (4), we take $M_{G_0} = 2.56 \pm 0.13$ GeV [11] and assume $M_N = M_{\pi^0}$ [10,15], then M_S can be obtained from Gell-Mann-Okubo mass formula [16]

$$M_S^2 = 2M_K^2 - M_N^2, \tag{22}$$

where $M_K^2 = (M_{K^\pm}^2 + M_{K^0}^2)/2$, and $M_{K^\pm}$, M_{π^0} are the masses of pseudoscalar mesons $K^\pm$ and π^0, respectively. Apart from M_{G_0}, M_N, M_S and the masses of the observed mesons used in this paper (All the values of mass of the observed mesons used in this paper are taken from Particle Data Group 98 [9] except for $M_{\eta''} = 1416\pm2$ MeV [6]), we take the experimental data of Eqs. (9)$\sim$(16) [9] (see TABLE I) and $m_u/m_s = 0.642$ [17] as input. In this way, we use the 11 equations, (6)$\sim$(16), to determine the 4 unknown parameters in Eqs. (4), A_1, A_2, A_3 and r. The parameters are determined as $A_1 = 0.2493\ \mathrm{GeV}^2$, $A_2 = -0.2386\ \mathrm{GeV}^2$, $A_3 = -4.8105$ GeV^2 and $r = 2.9605$ with $\chi^2/d.o.f$(the χ^2 per degree of freedom)$= 1.99/7$. Based on the values of above parameters, the matrix M^2 remains hermitian, and from Eqs. (3) and (4), the unitary matrix U can be given by

$$U = \begin{pmatrix} x_{\eta''} & y_{\eta''} & z_{\eta''} \\ x_{\eta'} & y_{\eta'} & z_{\eta'} \\ x_{\eta} & y_{\eta} & z_{\eta} \end{pmatrix} = \begin{pmatrix} 0.3879 & 0.2924 & -0.8741 \\ -0.5693 & -0.6698 & -0.4766 \\ 0.7249 & -0.6825 & 0.0933 \end{pmatrix}. \tag{23}$$

From Eq. (5), the physical states η, η' and η'' can be read as

$$\begin{aligned} |\eta''\rangle &= 0.3879|N\rangle + 0.2924|S\rangle - 0.8741|G_0\rangle, \\ |\eta'\rangle &= -0.5693|N\rangle - 0.6698|S\rangle - 0.4766|G_0\rangle, \\ |\eta\rangle &= 0.7249|N\rangle - 0.6825|S\rangle + 0.0933|G_0\rangle. \end{aligned} \tag{24}$$

The fit results of Eqs. (9)$\sim$(21) are shown in TABLE I.

Eq. (24) shows that η'' (η', η) contains about 15% (32.4%, 52.5%) $(u\bar{u} + d\bar{d})/\sqrt{2}$ component, 8.5% (44.9%, 46.6%) $s\bar{s}$ component and 76.5% (22.7%, 0.9%) glueball component,

which supports the argument that η'' is a mixed $q\bar{q}$ glueball having a large glueball component [7,8]. Eq. (24) also shows that the interference between $|N\rangle$ and $|S\rangle$ is constructive for η'' and η' while destructive for η and that the interference between $|S\rangle$ and $|G_0\rangle$ is destructive for η'' and η while constructive for η'. Furthermore, the value of A_3 shows that fermion-loop corrections to the mass of the pseudoscalar glueball obtained in quenched approximation is quite large, which disagrees with that the quenched prediction agrees with the full QCD (unquenched) value to within 10% [18].

IV. SUMMARY AND CONCLUSIONS

We modify Kawai model and re-investigate the mixing of η, η' and η'' based on the modified model. The glueball-quarkonia content of the three states is determined from a fit to the data of the electromagnetic decays involving η, η'. Some predictions about the electromagnetic decays involving $\eta(1410)$ are presented. Our conclusions are as follows:

1). In the presence of $A_3 = A_2^2/A_1$, in order to make the matrix M^2 remain hermitian, the mass of the pseudoscalar glueball in quenched approximation, M_{G_0}, would be less than 2 GeV, which is inconsistent with the prediction given by lattice QCD in quenched approximation. However, in the absence of $A_3 = A_2^2/A_1$, not only can M_{G_0} be related to the prediction given by lattice QCD in quenched approximation but also the matrix M^2 remains hermitian.

2). η is dominantly a $q\bar{q}$ meson (52.5% $(u\bar{u}+d\bar{d})/\sqrt{2}$ and 46.6% $s\bar{s}$) with a negligible glueball component (0.9%). η' is dominantly a $q\bar{q}$ meson (32.4% $(u\bar{u}+d\bar{d})/\sqrt{2}$ and 44.9% $s\bar{s}$) with a quite large admixture of glueball (22.7%). η'' is dominantly a glueball (76.5%) with a admixture of $(u\bar{u}+d\bar{d})/\sqrt{2}$ (15%) and $s\bar{s}$ (8.5%).

3). The interference between $|N\rangle$ and $|S\rangle$ is constructive for η'' and η' while destructive for η. The interference between $|S\rangle$ and $|G_0\rangle$ is destructive for η'' and η while constructive for η'.

4). For the mass of the pseudoscalar glueball, the fermion-loop corrections to the predic-

tion given by lattice QCD in quenched approximation is quite large, which disagrees with the argument that the quenched prediction agrees with the full QCD (unquenched) value to within 10% [18].

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REFERENCES

[1] L.Kopke and N.Wermes, Phys. Rep. **174**, 67 (1989).

[2] S.Godfrey and J.Napolitano, Rev.Mod.Phys.**71**, 1411 (1999).

[3] T.Barnes, F.E.Close, P.Page and E.Swanson, Phys.Rev.**D55**, 4157 (1997).

[4] J.Augustin et al., Phys.Rev.**D42**, 10 (1990).

[5] Z.Bai et al., Phys.Rev.Lett.**65**, 2507 (1990).

[6] A.Bertin et al., Phys.Lett.**B361**, 361 (1995).

[7] F.E.Close, G.R.Farrar and Z.Li, Phys.Rev.**D55**, 5749 (1997).

[8] I.Kitamura, N.Morisita and T.Teshima, Int.J.Mod.Phys.**A31**, 5489 (1994).

[9] Particle Data Group (C.Caso *et al.*,) Eur.Phys.J.**C3**, 1 (1998).

[10] E.Kawai, Phys.Lett.**B124**, 262 (1983).

[11] C.J.Morningstar and M.Peardon, Phys.Rev.**D60**, 034509 (1999).

[12] A.Seiden, H.Sadrozinski and H.E.Haber, Phys.Rev.**D38**, 824 (1988).

[13] H.E.Haber and J.Perrier, Phys.Rev.**D32**, 2961 (1985).

[14] F.E.Close, An introduction to quarks and partons (Academic Press, London, 1979).

[15] T.Feldmann, P.Kroll and B.Stech, Phys.Rev. **D58**, 114006 (1998).

[16] S.Okubo, Prog.Theor.Phys.**27**, 949 (1962).

[17] J.Rosner, Phys.Rev.**D27**, 1101 (1983).

[18] S.Aoki et al., Nucl.Phys.Proc.Suppl. **73**, 189 (1999).

TABLES

Decay Modes	Fit	Exp. [9]	Decay Modes	Fit	Exp. [9]
$\frac{\Gamma(\eta\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}$	52.36	58.46 ± 9.03	$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}$	571.07	540.78 ± 104.44
$\frac{\Gamma(\eta''\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}$	709.90		$\frac{\Gamma(\rho\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)}$	0.067	0.051 ± 0.023
$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)}$	0.087	0.086 ± 0.016	$\frac{\Gamma(\eta''\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)}$	1.025	
$\frac{\Gamma(\eta''\to\omega\gamma)}{\Gamma(\omega\to\pi^0\gamma)}$	0.110		$\frac{\Gamma(\eta''\to\phi\gamma)}{\Gamma(\omega\to\pi^0\gamma)}$	0.011	
$\frac{\Gamma(\phi\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)}$	0.075	0.078 ± 0.010	$\frac{\Gamma(\phi\to\eta'\gamma)}{\Gamma(\omega\to\pi^0\gamma)}$	0.0003	0.0007 ± 0.0005
$\frac{\Gamma(J/\psi\to\rho\eta)}{\Gamma(J/\psi\to\omega\pi^0)}$	0.474	0.460 ± 0.120	$\frac{\Gamma(J/\psi\to\rho\eta')}{\Gamma(J/\psi\to\omega\pi^0)}$	0.226	0.250 ± 0.079
$\frac{\Gamma(J/\psi\to\rho\eta'')}{\Gamma(J/\psi\to\omega\pi^0)}$	0.061				

TABLE I. The fit results as well as the experimental data of the electromagnetic decays involving η, η' and η''.